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A MODEL FOR STRATEGIC VOTING*

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Abstract. A model combining an individual voter's personal preferences (a cardinal utility rating) with the preferences of the rest of the electorate (as measured by pre-election polls) is investigated to determine how a voter may maximize his expected satisfaction with the election outcome. The basic model of McKelvey and Ordeshook for a one man-one vote election procedure is generalized to one man-N votes, approval voting and committee elections. Explicit decision conditions are derived for each procedure. An approximation technique is described and some numerical results are included. The final section extends the basic deterministic model to a situation in which the optimal voting strategy may be probabilistic. The model can be used to compare voting procedures as well as the strategies available to a voter under a particular voting procedure.

1. Introduction. The usual "one man-one vote" election system has serious drawbacks when the number of candidates exceeds two. A voter who always votes for his favorite candidate may forfeit much of his electoral power if his favorite candidate has little chance of winning. The voter who is willing to vote for a candidate other than his favorite still has the problem of deciding for whom to vote.

Several investigators have recently examined and compared various election systems for multicandidate elections. Brams [4], Fishburn [10], Lucas [15], Merrill [18] and Weber [22] concentrated on examining the properties of the systems inherent in the rules for the system and showed that one or another voting system is "better" in certain respects. We take a different approach and concentrate on the individual voter rather than the system.

In this paper we investigate how the individual voter may maximize his power by strategically using his vote(s) in the optimal manner allowed by the election system used. By considering both the individual voter's personal preferences (a cardinal utility rating of the candidates) as well as the preferences of the rest of the electorate (as measured by pre-election polls), the model enables the voter to determine for whom to vote in order to maximize his expected satisfaction (utility) with the election outcome. The general model will facilitate the further comparison of election systems by providing a unified framework so that the optimal voting behavior of voters can be compared. It can also be of use to the individual voter in a multicandidate election and some sample calculations are included.

A utility model for 2- and 3-candidate elections was first developed by McKelvey and Ordeshook [16] for elections in which each voter had one vote. We extend the model to elections with possibly more than one winner (committees) and in which each voter may have more than one vote, including approval voting elections. Methods for approximating the necessary numerical conditions for the model are suggested and numerical results are given for some elections. The calculated optimal voting behavior appears to agree well with the intuitive optimal behavior of voters.

2. Assumptions and notation. Throughout this analysis the term "the voter" will refer to that individual voter whose optimal course of voting action will be determined. Two basic pieces of information about the voter and the rest of the electorate will be needed: (1) It is assumed that the voter has a cardinal personal preference scale on which he rates his expected satisfaction with each of the possible election outcomes;

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and (2) It is assumed that some information about the preferences of the rest of the voting population is known. The term "the voter's rating of candidate A" will mean the voter's expected satisfaction with the election result if candidate A is elected. The information about the preferences of the rest of the electorate would most commonly come from pre-election polls, but could be the result of educated guesses. It is supposed that the electorate's preferences are known only imperfectly, and an estimate of the accuracy of this information is used in the analysis.

The two pieces of information used in the model are the ones which create conflict for the concerned voter. In this analysis the concerned voter will attempt to resolve the possible conflict between his preferences and those of the rest of the electorate by voting in the way which is most likely to increase his satisfaction with the election outcome.

Let N be the number of candidates in the election, and let C_i = the ith candidate, R_i = the voter's personal rating of C_i, W_i = the probability that C_i wins the election and T_{ij} = the probability that C_i and C_j tie for first ($i \neq j$). The values of R_i and T_{ij} will determine the voter's course of action in the voting systems analyzed. The values of the T_{ij} are crucial to this analysis, and the model includes an explicit means of calculating them. Since the probabilities of three-way ties for first place are so much smaller than those for two-way ties, they will have negligible influence on the voter's decisions and are not used here. It is supposed in elections without runoffs that if two candidates are tied for first, the winner will be decided in a fair manner so each of the tied candidates is equally likely to win.

If C_i wins the election, the voter obtains an expected satisfaction (utility) of R_i, and if C_i and C_j tie for first, the voter will obtain an expected satisfaction of $(R_i + R_j)/2$. Thus, the overall expected satisfaction for the voter is

$$\sum_i W_i R_i + \sum_{i \neq j} T_{ij} \frac{R_i + R_j}{2}. \tag{1}$$

Rather than recalculate the total expected satisfaction for the voter for every possible vote, the model will maximize the voter's expected gain in satisfaction. If (1) is the voter's expected satisfaction before the voter votes and the voter casts 1 vote for C_1, then a gain is achieved in two possible ways: (i) C_1 had been losing but is now tied for first with C_j, or (ii) C_1 had been tied for first place with C_j but now C_1 wins. If it is assumed that the probability of losing by one vote is the same as the probability of a tie (losing by 0 votes), then the expected gain in each case is $T_{1j}(R_1 - R_j)/2$. Hence, a vote for C_1 gives the voter a total expected gain in satisfaction of $T_{12}(R_1 - R_2) + T_{13}(R_1 - R_3) + \cdots + T_{1N}(R_1 - R_N)$.

3. The geometry of an election. Calculating T_{ij}. In a 3 candidate election, the set of election outcomes is $\{(v_1, v_2, v_3)$: v_i = number of votes cast for C_i and $v_1 + v_2 + v_3 = N$, the total number of votes cast by the electorate$\}$. Each of the possible outcomes can be represented as a 2-dimensional outcome triangle by converting to barycentric coordinates (Fig. 1). A_{ij} is that portion of the outcome triangle in which C_i loses to C_j by one vote or less; it is the region within 1 unit (vote) of the line $v_i = v_j > v_k$ on the C_j side of the line. T_{ij} is the probability that the election result (v_1, v_2, v_3) lies in the region A_{ij}.

McKelvey and Ordeshook [16] only applied their model to 2- and 3-candidate elections in which each voter had 1 vote, and they limited their numerical calculations to a uniform probability distribution. We shall extend their work in several directions: M candidate elections, elections of committees, elections in which a voter has N votes

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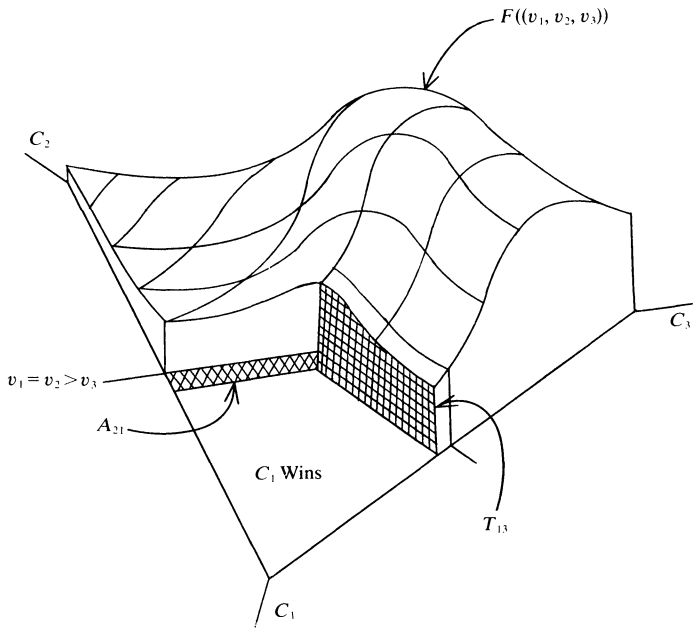

FIG. 1. *The geometry of a* 3*-candidate election.*

and "approval voting" elections. Numerical results for a Gaussian distribution will be given.

Let E_i be the percentage of the vote predicted for C_i by the pre-election polls. We construct a Gaussian distribution F on the outcome triangle by $F((v_1, v_2, v_3)) = K \cdot \exp(-(1/2)D^2/S^2)$, where D is the distance from (E_1, E_2, E_3) to $(v_1, v_2, v_3)/N$ and S^2 is a measure of the uncertainty in the prediction. A Dirichlet distribution may be more appropriate because of the nonzero correlations between the v_i, but we use the Gaussian for simplicity. The necessary computation is the evaluation of the integral

$$T_{ij} = \iint_{A_{ij}} F((v_1, v_2, v_3)).$$

These volume calculations can be greatly simplified by observing that each region A_{ij} is very narrow, only one vote wide. Thus, T_{ij} will be (almost) proportional to the area of the face of T_{ij}, and we will only need the ratios of T_{ij}'s. These areas can then be numerically approximated using Simpson's rule or Romberg integration.

For a 4-candidate election, the outcome space will be a solid tetrahedron. The areas of election uncertainty A_{ij} will be certain internal faces of the tetrahedron, and the T_{ij} can be approximated by approximate integration along a series of lines on each face. As the number of candidates increases, the geometry becomes more complicated, but the T_{ij} values can still be approximated. The model can also be used for elections in which winning is not the only goal. In some primaries it is important for psychological and financial reasons that a candidate not place too low. If the voter can assign utility values to nonfirst place outcomes, then the probabilities can be approximated and used to find an optimal voting strategy.

In the following analysis of particular voting systems, we will derive critical values A^* such that the voter should vote in one way if the rating of a candidate is greater than A^* and another way if the rating is less than A^*. These A^* will depend on the

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R_i, the T_{ij} and the voting system being considered. For a 3-candidate, one man-one vote election,

$$A^* = \frac{R_1(2T_{12}+T_{13})+R_3(T_{23}-T_{13})}{2T_{12}+T_{23}},$$

and the voter should vote for C_1 if $R_2 < A^*$ and for C_2 if $R_2 > A^*$. This type of weighted mean of the ratings of the candidates occurs for all of the voting systems examined. Figure 2 shows the critical values for a 3-candidate election with $S^2 = .05$,

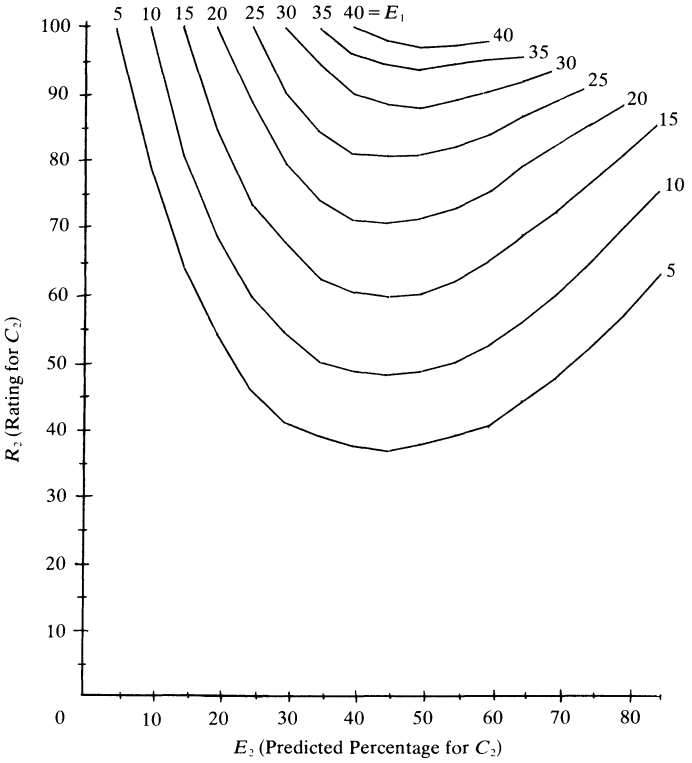

FIG. 2. *Critical values for the* 3*-candidate one man-one vote election* ($S^2 = .05$).

the ratings of the candidates normalized so $100 = R_1 \geqq R_2 \geqq R_3 = 0$ and the T_{ij} values approximated using Simpson's rule over 20 equally spaced points. If the pre-election poll predicts an outcome (E_1, E_2, E_3), then the voter should vote for C_2 if and only if the point (E_2, R_2) lies above the curve corresponding to E_1. Thus, if $(E_1, E_2, E_3) = (.25, .35, .40)$, then the voter should vote for C_2 if and only if $R_2 > 84$. Figure 3 contains the same information for $S^2 = .01$. If (E_1, E_2, E_3) is again $(.25, .35, .40)$, then the voter should vote for C_2 if and only if $R_2 > 53$. As it becomes more certain (as S^2 gets smaller) that the favorite C_1 will not win, the voter should be more likely to vote for C_2, i.e., A^* gets smaller. Even if one can not accept the precise nature of the quantitative results, the qualitative suggestions concerning whom to vote for should still be of value to a concerned voter.

4. Other voting procedures. The one man-one vote situation described is the most common multicandidate procedure and appears frequently in primary elections.

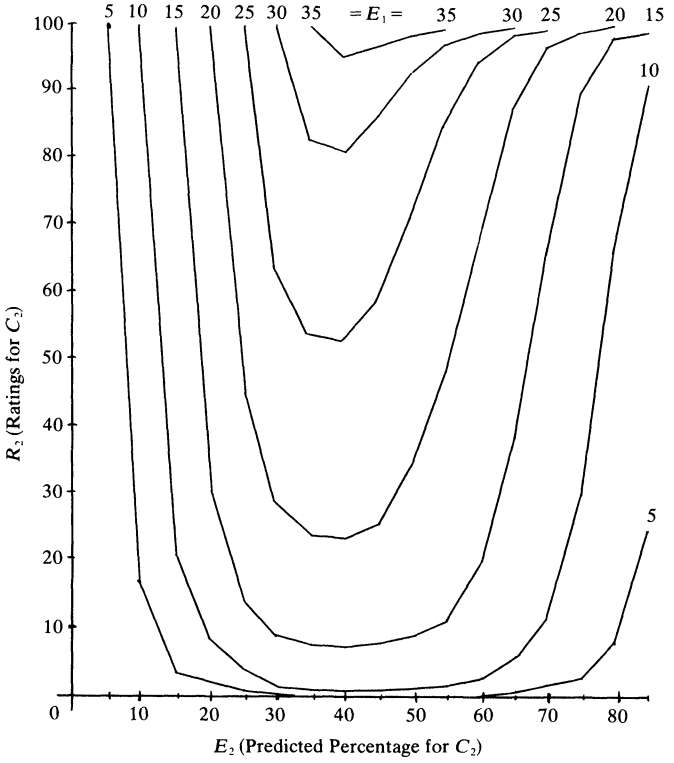

FIG. 3. *Critical values for the 3-candidate one man-one vote election* ($S^2 = .01$).

The remainder of this analysis will determine optimal voting strategies for elections involving multiple votes per voter, including approval voting and multiple winner elections (committees).

Occasionally an election is held in which each voter has $N > 1$ votes. Generally this occurs for committee elections, but we first examine it for a single winner election. We will consider the cases where the voter may (i) cast all of his votes for a single candidate and (ii) cast only 1 vote for each candidate. Assume throughout that $R_1 \geqq R_2 \geqq \cdots \geqq R_M$, where M is the number of candidates. We also assume that the probability that C_i loses to C_j by 1 vote is approximately the same as the probability that C_i loses to C_j by exactly n votes, $1 \leqq n \leqq N$. As long as the total votes cast by the electorate is large compared to the N votes available to a single voter this assumption is reasonable, and it greatly simplifies the calculations.

In the unrestricted case, the voter is allowed to distribute his N votes among the M candidates as he sees fit. Let $G(v_1, v_2, \cdots, v_M)$ be the expected gain in utility from casting v_i votes for C_i, $\sum v_i = N$. By selecting $p_i = v_i/N$, $p = (p_1, p_2, \cdots, p_M)$ is an M-dimensional probability vector, and the set P of all such probability vectors is a convex compact set.

$$G(v_1, \cdots, v_M) = G(p_1 N, \cdots, p_M N) = \sum_{i=1}^{M-1} \sum_{j=1}^{M} (p_i - p_j) \cdot N \cdot T_{ij} \cdot (R_i - R_j)$$

is a linear function of the variables p_i on the set P, so G attains its maximum at an extreme point of P. But these extreme points are the unit basis vectors of R^M, and each corresponds to casting all N votes for a single candidate. Thus, it is never uniquely

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optimal for a voter to split his N votes among two or more candidates. In the case of 3 candidates ($M = 3$), the explicit decision criteria is

$$B^* = \frac{R_1(2T_{12} + T_{13}) + R_3(T_{23} - T_{13})}{2T_{12} + T_{23}}.$$

If $R_2 > B^*$, then the optimal strategy is to cast all N votes for C_2. If $R_2 < B^*$, then the optimal strategy is to cast all N votes for C_1. B^* is independent of the number of votes N which the voter has as long as the original assumption holds.

Occasionally a voter has $N > 1$ votes but is not allowed to cast more than $K < N$ of them for a single candidate. This imposes the extra constraint on the p_i in the previous paragraph that each $p_i \leqq K/N$, but the set of such probability vectors is still compact and convex, so the previous argument again shows that the voter should "lump" his votes as much as is allowed by the election rules. In this situation, it may be optimal for the voter not to use all of his votes as will be the case in an approval voting election.

5. Approval voting. Approval voting is a recently developed (1976) voting procedure in which each voter may cast 1 vote for as many or as few of the candidates as the voter "approves of"; that is, a voter may cast only one vote for a candidate but may vote for as many candidates as desired. It is being extensively examined (see [4]–[6], [11]–[14], [17], [19], [20], [22]) because it appears to have such desirable properties. Fishburn and Brams [12] show that if a strict Condorcet winner must be elected in a one man-one vote election when the voters use admissible strategies, then he must also be elected under approval voting if the voters use admissible strategies; but the converse is not true. Merrill [17], [19] has contrasted approval voting with several other voting procedures in terms of the relative difficulty for the voter to approximate his optimal strategies. Fishburn and Brams [13] also investigated questions of "power" and "efficacy", and Hoffman [14] measured "cost" and "relative efficiency". The main decision critera C_i^* is independently due to Merrill [19] and Hoffman [14] and is given here for the sake of completeness.

For N candidates, a straightforward argument implies that the voter should always vote for C_1, never vote for C_N and should vote for C_i, $1 < i < N$, if and only if $R_i > C_i^*$, where

$$C_i^* = \sum_{\substack{j=1 \\ j \neq i}}^{N} R_j T_{ij} \Big/ \sum_{\substack{j=1 \\ j \neq i}}^{N} T_{ij}.$$

So in an approval voting election, the voter should vote for C_i if and only if R_i is greater than a certain weighted mean of the ratings of the other candidates. If all of the candidates are deemed equally likely to win, then all of the T_{ij} are equal and C_i^* reduces to

$$C = \sum_{j=1}^{N} \frac{R_j}{N},$$

which is independent of i. The decision value C is due to Merrill [17] and Weber [22].

In the equally likely case, it is clear that the optimal voting strategy given by the model is "sincere" in the sense that if $R_k > C$, then $R_i \geqq R_k > C$ for all $i < k$. So, if the optimal strategy for the voter includes a vote for C_k, then it will also include votes for all higher rated candidates. In the nonequally likely case, it is shown in [6] that the optimal strategy is sincere for $N = 3$. For $N \geqq 4$, the optimal strategy may not be

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sincere, but it can be shown [14] that the optimal sincere strategy is almost as good as the optimal strategy.

The techniques of § 3 can be used to calculate the values for C_i^*. Figure 4 shows the C^* values for a 3-candidate approval voting election with ratings normalized to $100 = R_1 \geqq R_2 \geqq R_3 = 0$. Simpson's rule was used over 20 equally spaced points in the calculations. If the pre-election poll predicts an outcome of (E_1, E_2, E_3) with $S^2 = 0.01$, then the voter should include a vote for C_2 if and only if the point (E_2, R_2) lies above the curve corresponding to the E_1 value. Thus, if $(E_1, E_2, E_3) = (.25, .35, .40)$, the voter should include a vote for C_2 if and only if $R_2 > 19$. As it becomes more likely that C_1 will not win, the voter should be more likely to include a vote for C_2, i.e., C^* gets smaller.

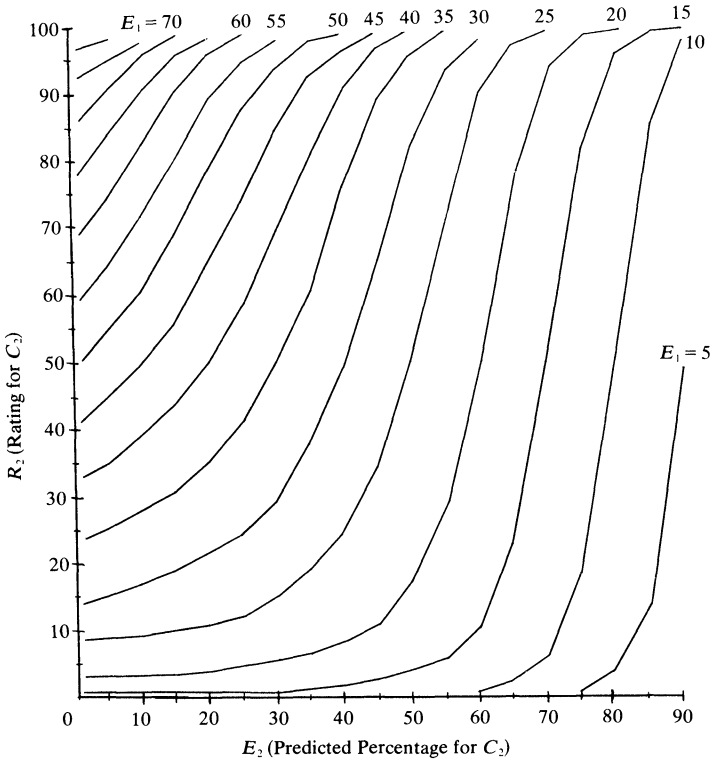

FIG. 4. *Critical values for the 3-candidate approval voting election* ($S^2 = .01$).

6. Electing a committee. The analysis of single winner elections can be extended to several winner elections, i.e., committees. The use of a probability distribution defined on the outcome space is the same as in § 3, but the regions of interest are different. We first consider the election of a committee of 2 from a slate of 3 candidates.

The voter again must assign an expected utility to each of the possible outcomes (i.e., committees) in some reasonable way. The voter could (i) assign a value to each committee as a unit or (ii) assign a value to each candidate and the sum of the values of the numbers to the committee. Assignment system (i) is a more realistic measure, but it would be very tedious with more than 3 or 4 candidates. The analysis preceeds independently of the rating system in use.

Denote the three possible committees $x = \{C_1, C_2\}$, $y = \{C_1, C_3\}$, $z = \{C_2, C_3\}$. Let R_k be the rating assigned to committee k, $k = x$, y or z. Let A_{xy} be the area of

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indeterminacy between regions x and y, and let T_{xy} be the probability that the election outcome falls in region A_{xy}. T_{xy} can be approximated using the techniques of § 3. It is assumed that the probability that C_i and C_j tie is (approximately) the same as the probability that C_i loses to C_j by 1 vote. If the voter does not vote, then his expected satisfaction with the election outcome will be

$$W_xR_x + W_yR_y + W_zR_z + T_{xy}\left(\frac{R_x+R_y}{2}\right) + T_{xz}\left(\frac{R_x+R_z}{2}\right) + T_{yz}\left(\frac{R_y+R_z}{2}\right),$$

where W_k is the probability that committee k is elected. We assume that the committees are labelled so that $R_x \geqq R_y \geqq R_z$.

For a one man-one vote election let

$$D^* = \frac{R_x(T_{xy}-T_{xz}) + R_z(2T_{yz}+T_{xz})}{T_{xy}+2T_{yz}}.$$

If $R_y > D^*$, then $G(1, 0, 0) > G(0, 1, 0)$, so a vote for C_1 is optimal. If $R_y < D^*$, then $G(1, 0, 0) < G(0, 1, 0)$, so a vote for C_2 is optimal.

An interesting situation commonly occurs in the election of a committee: the voter's favorite candidate is also the consensus favorite and is almost certain to be elected to the committee. Should the voter forego voting for his favorite in this case and try to get his second choice onto the committee? Both intuitively and according to the model the answer is sometimes "yes": if the favorite has a strong lead and if the second favorite has a reasonable chance of defeating the third candidate. Let $100 = R_x \geqq R_y \geqq R_z = 0$, and suppose $T_{xy} = 0.1$ and $T_{xz} = T_{yz} = 0.01$. Then the voter should vote for C_2 if $R_y < 75$, i.e., if committee $y = \{C_1, C_3\}$ is sufficiently undesirable. If C_1 increases his lead so $T_{xy} = 0.12$ and $T_{xz} = T_{yz} = 0.005$, then the voter should vote for C_2 if $R_y < 88.5$.

For a one man-N vote election, we will again assume that the total number of votes cast is large compared to N (see § 4). For $0 < a < 1$, $G((1-a)N, aN, 0) = T_{xz}(R_x - R_z)(1-a)N + T_{yz}(R_y - R_z)(1-2a)N + T_{xy}(R_x - R_y)(a)N$. If $R_y > D^*$, then $G(N, 0, 0) \geqq G((1-a)N, aN, 0) \geqq G(0, N, 0)$ for $0 < a < 1$, so the voter should cast all N votes for C_1. If $R_y < D^*$, then the inequalities are reversed and the voter should cast all N votes for C_2. The voter should never vote for C_3, and it is never uniquely optimal to split votes among candidates. In the general M-candidate case, an argument similar to that of § 4 shows that it is still never uniquely optimal for a voter to distribute his N votes over several candidates.

Brams [3] has examined the election of several candidates, a committee, by voters who vote in blocs. If some of the blocs have sufficient votes to guarantee the election of some candidates, he showed that those blocs should spread their votes over an optimal number of candidates. The spreading or grouping of votes thus depends on which assumption, if either, holds.

If a voter is only allowed to cast a maximum of $N/2$ votes for a single candidate, the election is equivalent to an approval voting election with 3 candidates. Let $E^* = (R_xT_{xy} + R_zT_{yz})/(T_{xy} + T_{yz})$. If $R_y > E^*$, then the voter should vote $(N/2, 0, 0)$. If $R_y < E^*$, he should vote $(N/2, N/2, 0)$.

7. A modification. The foregoing analysis tacitly assumed that the voter was the only member of the electorate employing the model to determine for whom to vote. In most elections, especially those with a large number of voters, this assumption will

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cause no problems since few persons are likely to be willing and able to go through the necessary computations. However, by making the model slightly more complex it is possible to eliminate the assumption. But the revised model will require more information about the electorate, and the final decision will now be probabilistic.

Consider an election to elect 2 of 3 candidates and in which each voter has 1 vote. Suppose that the various blocs of the electorate rate the candidates as in Table 1. If each voter votes for his favorite, then the outcome will be C_1 with 60%, C_2 with

TABLE 1
An example.

Electoral group	% of the total electorate	Ratings (R_1, R_2, R_3)
I	40	(100, 80, 10)
II	20	(100, 20, 75)
III	21	(50, 50, 100)
IV	19	(61, 100, 60)

19% and C_3 with 21%, so the committee will consist of C_1 and C_3. However, if all of the voters use the basic model, then the backers of C_2 and C_3 will still vote for C_2 and C_3, since those candidates are in a close race for the second position on the committee. But the backers of C_1, seeing that their candidate is "assured" of winning, will vote for C_2 or C_3 and attempt to break the virtual tie between those candidates. If all of the voters use the basic model, the outcome will be C_1 with 0 votes, C_2 with 59% and C_3 with 41%, so C_2 and C_3 will be elected even though 60% of the voters think C_1 is the best candidate.

Since it is unrealistic in a large electorate to suppose that 60% of the voters could get together and decide which of them will vote for C_1 to insure his election and which will vote for the other candidates, we resort to probabilistic considerations. A backer of C_1 will with probability q use the basic model and vote for C_2 or C_3 and will with probability $(1-q)$ vote for C_1. In this way C_1 can be "assured" of election and the "excess" C_1 votes can be used to decide between C_2 and C_3. This revised method will not require collaboration among the C_1 backers.

Let F = the voter's favorite candidate who is also the favorite of the electorate; V = the percentage of the electorate for whom F is the favorite; W = the percentage of the total vote needed for a candidate to be assured of election ($W = 1/3$ in the previous example) and P = the percentage of the backers of F who will use this revised model. P is the additional information required for this revised model. Put $q = \min\{1, (1/P)-(W/(PV))\}$. A user of this revised model should:

(i) use the basic model (not vote for F) with probability q,

(ii) vote for F with probability $(1-q)$.

If each user of the revised model makes the above determination independently, then the expected percentage of votes for F will be (percentage of voters who favor F and do not use the revised model) + (percentage of voters who favor F and use the revised model and still vote for F) $= V(1-P)+V(P)(1-q) = V - VPq = V - VP((1/P)-(W/(PV))) = W$ and W is sufficient to insure the election of F.

In the example of this section, $V = 0.6$, $W = 1/3$ and 59% of the electorate think the best committee would be C_1 (or F) and C_2. If 40% of the backers of F (groups I and II) use the revised model, then $P = 0.4$ and $q = \min\{1, 1.1\} = 1$, so the 40% of

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the backers of F who use the revised model should (i) not vote for F with probability 1, and (ii) vote for F with probability 0. Even if $P = 1$, then $q = \min\{1, 0.44\} = 0.44$, so the user of the revised model should (i) not vote for F with probability 0.44 and (ii) vote for F with probability 0.56.

The revised model calls for a deviation from the basic model only if a relatively large proportion of the backers of the favorite are expected to use the model. It seems unlikely that such a deviation will be necessary unless the electorate is small or unusually sophisticated.

8. Extensions and further work. The model extends naturally to the election of a single candidate or committee from a slate of N candidates—all that is needed is a probability distribution defined on the $(N-1)$-dimensional outcome space and an expected satisfaction rating assigned to each of the possible outcomes. The ideas and techniques also extend to the analysis of multistage elections. Suppose that the election rules in a 3-candidate single winner election require a runoff election if no candidate receives a majority of the votes on the first ballot. The outcome regions for these rules are shown in Fig. 5. The voter must now assign utility ratings to 6 possible outcomes—3 first-ballot win regions and 3 runoff regions. Once this assignment is made and once a probability distribution is imposed on the outcome space, the probabilities that the outcome is close to a boundary can be approximated. These multistage election models quickly become complicated but should yield reasonable results.

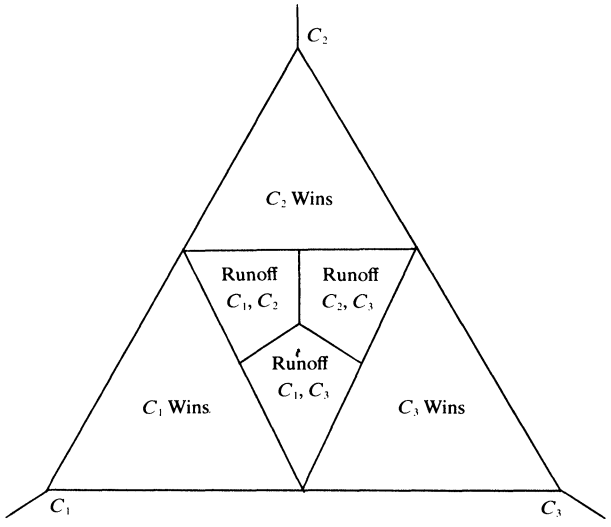

FIG. 5. *Outcome space for a* 3*-candidate* 1*-winner election with runoff provision.*

An exact personal rating of utility may be difficult to assign, but the truly concerned voter should be able to assign ratings in some reasonable fashion. Then the resulting recommendations can be viewed qualitatively rather than quantitatively.

The use of pre-election polls to determine how to vote is certainly open to criticism and possible abuse. A numerical analysis of the effects of changes in the poll results on the various conditions would be appropriate. But any voting decision model which expects to maximize a voter's effect must somehow take into account the views of the rest of the electorate.

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